Soil–Foundation–Structure Interaction

**PART A**

**TYPES of FOUNDATION**

- **PILES**
  \[ \frac{L}{d} > 7 \]

- **CAISSONS**
  \[ \frac{D}{B} \approx 1 - 5 \]

- **FOOTINGS**
  \[ \frac{D}{B} \leq 1 \]
INTRODUCTION

to

Soil–Foundation

Dynamics.

Basic Concepts

\[
\begin{align*}
F_1/\delta_1 & \equiv K_v \quad (\text{Vertical}) \\
F_2/\delta_2 & \equiv K_{H,x} \\
F_3/\delta_3 & \equiv K_{H,y} \\
M_5/\theta_5 & \equiv K_{\theta,rx} \\
M_4/\theta_4 & \equiv K_{\theta,ry} \\
M_6/\theta_6 & \equiv K_t \quad (\text{Torsion})
\end{align*}
\]
HORIZONTAL and ROCKING Stiffness

Foundation Plan

\[ \mathcal{K}_H = \frac{Q}{u_H} \]
\[ \mathcal{K}_R = \frac{M}{\theta} \]

Foundation–Soil Section

\[ K = \frac{Q}{u} \]
\[ R = \frac{M}{\theta} \]

Springs + Dashpots: Physical Interpretation (Vertical Mode)

\[ P_z(t) = P_z e^{i \omega t} \]

\[ u_z(t) = u_z e^{i \omega t} \]

\[ P_z e^{i \omega t} = \bar{K}_z u_z + C_z \dot{u}_z \]

\[ u_z(t) = u_z e^{i \omega t} \]

\[ P_z e^{i \omega t} = \bar{K}_z u_z e^{i \omega t} + i \omega C_z u_z e^{i \omega t} \]
**Springs + Dashpots: Physical Interpretation (Vertical Mode)**

\[ P_z(t) = P_z e^{i\omega t} \]

\[ u_z = \frac{P_z}{K_z + i\omega C_z} \]

**Complex Dynamic Stiffness**

\[ \mathcal{K} = K_z + i\omega C_z \]

\[ K_z = K_z \& (\omega) \]

**Definition of Stiffness and Damping of a Foundation – Soil System**

\[ K_H = \frac{Q_o}{u_o} = \frac{8}{2-v} \text{GR} \quad , \quad C_H \propto V_s \text{R}^2 \]

\[ K_R = \frac{M_o}{r_o} = \frac{8(1-v)}{3(1-v)} \text{GR}^3 \quad , \quad C_R \propto V_{la} \text{R}^4 \]

\[ K_V = \frac{N_o}{W_o} = \frac{4}{1-v} \text{GR} \quad , \quad C_V \propto V_{la} \text{R}^2 \]
The recognized effects of SS on rigid and compliant ground are shown. The dynamic stiffness coefficient is plotted against the dimensionless frequency parameter $a_0 = \frac{\omega B}{V_s}$. The figures illustrate the behavior of $k_z$ and $k_y$ for different values of $L/B$ and $\nu$. Fine saturated soils with $\nu = 0.5$ are considered.

For a rigid base ($K$, $\beta$), the recognized effects are $\beta > \bar{\beta}$ and $T > \bar{T}$. For a compliant ground ($K_R$, $K_X$), the opposite effects are observed.

The diagrams show the comparison between the two cases, highlighting the differences in dynamic stiffness and frequency response.
\[ T_{st} = \frac{2\pi}{\sqrt{g}} \sqrt{\frac{W}{k_{st}}} \]

\[ \tilde{T} = \frac{2\pi}{\sqrt{g}} \sqrt{\Delta} \]

\[ \tilde{T} = T \left(1 + \frac{k}{K_H} + \frac{kh^2}{K_R} \right)^{1/2} \]

\[ \tilde{\beta} = \beta_{st} (\tilde{T}/T)^3 + f \left( \frac{h}{R}, \frac{\tilde{T}}{T}, \pi \rho \Omega^2 \right) \]

Fixed-base structure

Structure on flexible base

\[ \tilde{T} \gg T \]

\[ \tilde{\beta} > \beta \]
INTRODUCTION to Soil–Foundation–Structure Interaction.

Basic Concepts
Kinematic Loading

Inertial Loading
The PROBLEM

Soil – Pile – Structure Interaction
Kinematic Response

Ground Deforms

Inertial Response

\(mS_a\)
KINEMATIC RESPONSE:

*Interplay between pile and soil under seismic–wave motion of surrounding soil:*

Piles are stressed, by developing *Curvatures + moments*

*both at greater depths and the fixed head*

SEISMIC PILE BENDING

- inertial
- kinematic
- liquefaction induced
Kinematic Pile Moments:

\[ d = 1 \text{ m} \]

\[ V_1 = 120 \text{ m/s} \]

\[ V_2 = 480 \text{ m/s} \]

\[ \text{Aegion: } 0.54 \text{ g} \]

Soil Modelling and Methods of Analysis

- The soil as a continuum (usually elastic)
  - Analytical solutions \([ few \])
  - Numerical solutions: with Finite Elements, Boundary Elements, Hybrid Methods

- Soil reactions from independent Winkler Springs:
  - Analytical Solutions (for linearly-elastic springs)
  - Numerical solutions (for nonlinear and inelastic springs)

\[ \text{Widespread use of } p-y \text{ curves} \]
Analysis in 2 Steps

Ground Response

Soil – Pile Interaction
Example of comparison:
FEM vs. Winkler (BDWF).
Kinematic, Free–Head Pile

Pile Displacement: m

Pile bending moment: MNm
Simple **Crude** Expression for the KINEMATIC Bending Moment at the Interface of 2 layers (*V₁ ≠ V₂*)

\[
M \approx 0.042 \cdot \tau_c \cdot d^3 \cdot \left( \frac{L}{d} \right)^{0.30} \cdot \left( \frac{E_p}{E_1} \right)^{0.65} \cdot \left( \frac{V_2}{V_1} \right)^{0.50}
\]

Shear Stress at Interface \(\approx A_S \rho_1 H_1\)

*(Use with caution)*

Nikolaou & Gazetas 1997
3 Case Histories from Japan:

(1) Obha Ohashi Bridge Piles
(2) Konan High School Pile
(3) Ervic Building

KINEMATIC DISTRESS of an ACTUAL PILE in Futsisawa, Japan, during an Earthquake

Adapted from: Tazoh et al (1994)

• Ohba Ohashi Bridge Pier Foundations
• Heavily instrumented PILES (L = 26 m) in extremely SOFT organic clay
• Recorded numerous small and moderate motions
VALLEY of OHBA - HASHI
(Yokohama)

The Bridge: OHBA

V_s = 60 m/s
Modelling with Finite Elements (ABAQUS) and Spectral Elements (AHNSE)

The Ohba–Ohashi Bridge and the Instrumentation of Ground and Footing (Pier 6)

accelerographs
Ohba–Ohashi Bridge: Strain Gauges in two piles in Pier 6

L = 22 m

64 piles
(32 battered, 32 vertical)
Earthquake Records at Ohba-Ohashi Bridge

<table>
<thead>
<tr>
<th>M JMA</th>
<th>R</th>
<th>D</th>
<th>PGA [SURFACE]</th>
<th>PGA [BASE]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>238</td>
<td>10</td>
<td>1.9 % g</td>
<td>0.4 % g</td>
</tr>
<tr>
<td>6.0</td>
<td>81</td>
<td>70</td>
<td>3.1 % g</td>
<td>0.7 % g</td>
</tr>
<tr>
<td>6.0</td>
<td>42</td>
<td>20</td>
<td>11.4 % g</td>
<td>3.3 % g</td>
</tr>
<tr>
<td>5.4</td>
<td>38</td>
<td>20</td>
<td>2.6 % g</td>
<td>0.9 % g</td>
</tr>
</tbody>
</table>

M JMA = 6, R = 42 km
Evidently, the large bending strain at the depth of $\approx 22 \text{ m}$ is the result of the *sharp “impedance contrast”* at that location: interface between soft and very stiff soil.

As can be seen from the plot of next slide, it is purely the product of *kinematic* distress.
good performance of theory!

**active length** $\ell_c$ of a (flexible) pile

- **Definition**:

  - $L$: Total length of the pile
  - $\ell_c$: Active length of the pile
  - $d$: Diameter of the pile

  ![Diagram](image)
active length $l_c$ of a (flexible) pile

static loading [Gazetas 1991]

\[ l_c \approx 1.5d \left( \frac{E_p}{E_s} \right)^{0.25} \] ①

\[ l_c \approx 1.5d \left( \frac{E_p}{E_s^*} \right)^{0.22} \] ②

\[ l_c \approx 1.5d \left( \frac{E_p}{E_s^*} \right)^{0.20} \] ③

KINEMATIC DISTRESS of an ACTUAL PILE

in Hokkaido Japan during an Earthquake

Adapted from: Y. Miyamoto & K. Koyamada (2007)

- Tokachi-oki Earthquake 2003, M = 7.9
- Konan Junior High School (R = 240 km):
  - footings on piles
  - accelerograms: at 0 m and 153 m depth!!
TOKACHI-oki 2003 Earthquake:  M = 7.9 ....

KONAN High School

Accelerograph Array

50 m

High School
SOIL LAYERS and PHC (d=0.40 m) PILE

- Peat
- Clay
- Sandy
- Silt
- Sandstone

Shear wave velocity

$V_s \ [\text{m/s}]$

0 m 6 m 20 m 30 m

Clay

Sandy

Silt

Sandstone

0.35 g

0.05 g

153 m

ACCELEROMETERS

Recorded at Ground Surface

Approximately 0.35 g

Recorded at -153 m (“Input” Motion)

Approximately 0.05 g
Response Spectra (5% damping)

\[ S_a \]

± 0 m

0.35 g

- 153 m

0.05 g

\[ T \ (sec) \]

[Draw the analogy with the Mexico City 1985 spectra of the motions recorded at SCT (soil surface) versus UNAM (“rock” outcrop) !!]
Results of Analysis: ELASTIC PILE

\[ M / M_y \]

\[ M_y = \text{yield moment of pile section} \]

Indeed the pile was extracted from the ground, and a crack was seen at \( \approx 20 \text{ m} \) ! It was a complete crack, extending throughout the hollow cylindrical cross-section of the pile.

We thus would expect the pile to have been damaged at the depth of 20 m.
Sketch of observed cracks

GL-20.0 m
GL-20.3 m
GL-20.5 m

pile

crack

FIELD EVIDENCE
ERVIC BUILDING

Building accelerometer
Ground accelerometer
Dynamic strain transducer
CASE STUDY: Yokohama, Japan

EARTHQUAKE EVENT
Location – Tokyo Bay Area (N 35° 12'; E 139° 48')
Time – February 2, 1992
Magnitude – M = 5.9
Focal depth – D = 93 km
Epicentral distance – R = 32 km
Peak Acceleration – A = 0.05 g

ANALYSIS
Dominant Surface Acceleration Period – 1 sec
Fundamental Soil Period – 1 sec
Nonlinear effects – minor

Soil Profile & Transducers Locations

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Blowcounts</th>
<th>Shear Wave Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fill</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>Alluvial Sand</td>
<td>10</td>
<td>300</td>
</tr>
<tr>
<td>Alluvial Clay</td>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>Sand with Clay</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Tuffaceous Clay</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Clay with Sand</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Gravel</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Mudstone</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Fine Sand</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>Mudstone</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Interface 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interface 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_s = 1.6 \text{ Mg/m}^3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_s = 1.5 \text{ Mg/m}^3 )</td>
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</table>

09/05/2012
Measured / Predicted Spectra

X direction

Y direction

Measured / Predicted Pile Strains

peak pile bending strain (10^-5)

computed

measured corner pile

measured center pile
Inertial Response

Pile Head Fixity Conditions

“Fixed-Head” Piles

“Free-Head” Piles

θ_α = θ_ρ
(1) Transmitting the Shear Force, $Q_o$

[We ignore for the time being the (generally significant) pile-to-pile interaction]

Horizontal Force on each pile:

$$Q_i = \frac{Q_o}{n} \rightarrow 6$$

(2) Transmitting the Moment, $M_o$

Two mechanisms:

(a) Axial Forces

$$F_1 = (\theta_o \cdot x_i) \cdot K_V$$

$$M_{AA} = 2F_1x_1 + 2F_3x_3 =$$

$$= 2(\theta_o \cdot x_1 \cdot K_V) \cdot x_1 + 2(-\theta_o \cdot x_1 \cdot K_V) \cdot (-x_1) = 4 \cdot x_1^2 \cdot K_V \cdot \theta_o$$

(b) Bending Moments

$$M_{KP} = 6 \cdot M_1 = 6 \cdot K_M \cdot \theta_o$$
active length $\ell_c$ of a (flexible) pile
Active length $l_c$ of a (flexible) pile

Static Loading

$[\text{Gazetas 1991}]$

$\ell_c \approx 1.5 d \left( \frac{E_p}{E_s} \right)^{0.25}$  \hspace{1cm} (1)

$\ell_c \approx 1.5 d \left( \frac{E_p}{E_s^*} \right)^{0.22}$  \hspace{1cm} (2)

$\ell_c \approx 1.5 d \left( \frac{E_p}{E_s^*} \right)^{0.20}$  \hspace{1cm} (3)

(b) Characteristic Dimensionless Elastic Results:

STIFFNESS MATRIX at pile head

[for Flexible Piles]
**Homogeneous Layer**

\[ K_{HH} \equiv E_s d \left( \frac{E_p}{E_s} \right)^{0.21} \]

\[ K_{MM} \equiv 0.15 E_s d^3 \left( \frac{E_p}{E_s} \right)^{0.75} \]

\[ K_{MH} = K_{HM} \equiv -0.22 E_s d^2 \left( \frac{E_p}{E_s} \right)^{0.50} \]

---

**“Gibson” Soil:**

[ Gazetas 1991 ]

\[ K_{HH} \equiv 0.6 E_s d \left( \frac{E_p}{E_s^*} \right)^{0.35} \]

\[ K_{MM} \equiv 0.15 E_s d^3 \left( \frac{E_p}{E_s^*} \right) \]

\[ K_{MH} = K_{HM} \equiv -0.17 E_s d^2 \left( \frac{E_p}{E_s^*} \right) \]

---
**Pile – Pile Interaction**

*(Pile Group Effects)*
DYNAMIC STIFFNESS AND DAMPING OF FLOATING PILE GROUPS

3 x 3 PILE GROUP

DYNAMIC EFFICIENCY FACTOR

$K(\text{group})/9K(\text{one})$

STATIC Values
0.2 – 0.3

$\omega d/V_S$
But what about Nonlinear + Layerd Soil?

The Winkler Spring model offers perhaps the best available practical solution!

3 reasons for the particular success of Winkler model for piles (as compared with shallow fdns):

1) Theoretical:
2) Theoretical + Practical:

which $k_i$?
3) Practical: Availability of a wealth of nonlinear Winkler springs for a variety of soils, based on full-scale and laboratory experiments:

\[ p-y \text{ curves (non-linear springs)} \]

Two types of independent Winkler springs:

1. **Linearly Elastic Springs**

2. **Nonlinear Springs** (\( p-y \) curves)
The Elastic Winkler Model

REACTION of soil per unit length:

\[ p = p(y) = ky \]

\[ k = k_e \epsilon d \approx E_s \]

Lateral Winkler “Springs” + “Dashpots”

\[ k_x \approx 1.2E_s \]

\[ c_x \approx 6a_0^{-1/4}\rho_s V_s d + 2\beta_s \frac{k_x}{\omega} \]

Axial Winkler “Springs” + “Dashpots”

\[ k_s \approx 0.6E_s(1 + \frac{1}{2}\sqrt{a_0}) \]

\[ c_s \approx a_0^{-1/4}\pi d_\rho V_s \]
Lateral Pile Response *(displacements, moments)*:

**Winkler model**

**Free head pile:**

\[ y(z) = \frac{Fe^{-\lambda z} \cos \lambda z}{2EpIp \lambda^3} \]
\[ M(z) = -\frac{F}{\lambda} e^{-\lambda z} \sin \lambda z \]

**Fixed head pile (θ = 0):**

\[ \lambda = 4 \sqrt{\frac{k}{4Ep}} \]
\[ y(z) = \frac{Fe^{\lambda z}}{4EpIp \lambda^3} (\cos \lambda z + \sin \lambda z) \]
\[ M(z) = -\frac{F}{2\lambda} e^{-\lambda z} (\sin \lambda z - \cos \lambda z) \]

Resultant of soil tractions per unit length:

\[ P = \int (\sigma_r \cos \theta + \tau_{r\theta} \sin \theta) \, R \, d\theta \]
$p - y$ curves:

$$p_{\text{ult}} \geq \begin{cases} 9S_u d \\ 3K_p \gamma z d \end{cases}$$

**Ultimate Load**

Pile Slice: $p_{\text{ult}} \approx 9S_u D \div 12S_u D$

Surface Sq. Footing: $p_{\text{ult}} \approx 6S_u D$
3-D Passive Resistance of Limited Height Wall

\[ F_p = \mu p \left( \frac{1}{2} \gamma h^2 + q h \right) b K_p \]

\[ \mu_p \]

\[ h/b \]

\[ p - y \text{ curves:} \]

\[ p \geq \begin{cases} 9 S_u d \\ 3 K_P \gamma z d \end{cases} \]
Example: Elastic–Plastic Approximation of the $p$–$y$ curves
1. Nearly Elastic Response
2. Slightly Inelastic Response
3. Strongly Inelastic Response
Soil–Foundation–Structure Interaction

PART B
HORIZONTAL and ROCKING Stiffness

Foundation Plan

Foundation–Soil Section

\[ \mathcal{K}_H = \frac{Q}{u_H} \]

\[ \mathcal{K}_R = \frac{M}{\delta} \]
**Springs + Dashpots: Physical Interpretation (Vertical Mode)**

\[ P_z(t) = P_z e^{i \omega t} \]

\[ u_z = \frac{P_z}{\bar{K}_z + i \omega C_z} \]

\[ P_z e^{i \omega t} = \bar{K}_z u_z + C_z \dot{u}_z \]

\[ u_z(t) = u_z e^{i \omega t} \]

\[ P_z e^{i \omega t} = \bar{K}_z u_z e^{i \omega t} + i \omega C_z u_z e^{i \omega t} \]

**Complex Dynamic Stiffness**

\[ \bar{K} = \bar{K}_z + i \omega C_z \]

\[ \bar{K}_z = K_z \bar{K} (\omega) \]

**STATIC STIFFNESS**
For every mode of vibration: $\mathcal{K}_i$

**Complex Dynamic Stiffness**

$\mathcal{K}_z = \bar{K}_z + i \omega C_z$

$\bar{K}_z = K_z \kappa_z(\omega)$

*STATIC STIFFNESS*

$\kappa_z(\omega) = \text{Dynamic Stiffness Coefficient}$

**Complex Dynamic Stiffness**

$\mathcal{K}_i = \bar{K}_i + i \omega C_i$

$i = z, x, y, rx, ry, t$

---

**STATIC STIFFNESSES on HALFSPACE**

**CIRCLE, R**

- $K_v = \frac{4GR}{1-\nu}$
- $K_h = \frac{8GR}{2-\nu}$
- $K_\theta = \frac{9GR^3}{3(1-\nu)}$
- $K_t = \frac{16GR^3}{3}$

**STRIP, 2B**

- $K_v/L \approx \frac{1.2G}{1-\nu}$
- $K_h/L \approx \frac{2.1G}{1-\nu}$
- $K_\theta/L = \frac{\pi GB^2}{2(1-\nu)}$
### Arbitray Foundation Shape

<table>
<thead>
<tr>
<th>MODE</th>
<th>Static $K$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shape $(2B, 2L, A_b)$</td>
<td>SQUARE $(2B \times 2B)$</td>
</tr>
<tr>
<td>$z$</td>
<td>$K_z = \frac{2GL}{1-v} \left(0.73 + 1.54 \chi^{0.75}\right)$ $\chi = A_b/4L^2$</td>
<td>$K_z = \frac{4.54 GB}{1-v}$</td>
</tr>
<tr>
<td>$y$</td>
<td>$K_y = \frac{2GL}{2-\chi} 2 \chi$ $0.85$</td>
<td>$K_y = \frac{9GB}{2-v}$</td>
</tr>
<tr>
<td>$x$</td>
<td>$K_x = K_y \frac{0.2}{0.75} G L 1-B/L$</td>
<td>$K_x = K_y$</td>
</tr>
<tr>
<td>$rx$</td>
<td>$K_{rx} = \frac{G}{1-v} L \left[3 \left(\frac{L}{B}\right)^{0.15}\right]$</td>
<td>$K_{rx} = K_{rx}$</td>
</tr>
<tr>
<td>$ry$</td>
<td>$K_{ry} = \frac{G}{1-v} L \left[3 \left(\frac{L}{B}\right)^{0.15}\right]$</td>
<td>$K_{ry} = K_{rx}$</td>
</tr>
<tr>
<td>$t$</td>
<td>$K_t = G J_t^{0.75} [4 + 11 \left(1 - \frac{B}{L}\right)^{10}]$</td>
<td>$K_t = 8.3 GB^2$</td>
</tr>
</tbody>
</table>

### Dynamic Stiffness Coefficient

- **$k_z$**
  - Vertical
  - $\chi \leq 0.4$
  - $L/B = 1.2$

- **$k_y$**
  - Swaying ($y$)
  - $a_0 = \frac{\omega B}{V_s}$

- **$k_{ry}$**
  - Vertical
  - Fine saturated soils, $\chi = 0.5$

- **$k_{rx}$**
  - Vertical
  - $a_0 = \frac{\omega B}{V_s}$
Radius Dashpot Coefficients

\[ c_y = (\rho V_s A) \bar{c}_y \]  
\[ c_{rx} = (\rho V_l I) \bar{c}_{rx} \]

Surface Foundations
Homogeneous Halfspace

<table>
<thead>
<tr>
<th>Vibration Mode</th>
<th>Dynamic Stiffness ( \mathcal{K}(\omega) = K \cdot k(\omega) )</th>
<th>Radiation Coefficients ( C(\omega), \bar{C}(\omega) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical z</td>
<td>General shape ( K_x = \frac{G L}{L^2} \left( 0.73 + 1.54 x^{0.5} \right) ), ( x = \frac{a_0}{4 L} )</td>
<td>( C_y = (\rho V_s A) \bar{c}_y ) ( \bar{c}_y = \bar{c}_y \left( \frac{L}{B}, \frac{a_0}{V_s} \right) \to \text{Graph a} )</td>
</tr>
<tr>
<td>Horizontal y</td>
<td>General ( K_y = \frac{G L^2}{L^2} \left( 2 + 2.5 x^{0.5} \right) )</td>
<td>( k_y = k_y \left( L/B, a_0 \right) \to \text{Graph b} )</td>
</tr>
<tr>
<td>Horizontal x</td>
<td>General ( K_x = K_y \frac{G L (1 - B/L)}{L} )</td>
<td>( k_x = 1 )</td>
</tr>
<tr>
<td>Rocking rx</td>
<td>General ( K_{bx} = \frac{G L}{V_s A_b} \left( \frac{L}{B} \right)^{0.5} \left( \frac{V_s}{a_0} \right)^{0.5} )</td>
<td>( C_{rx} = (\rho V_s A) \bar{c}<em>{rx} ) ( \bar{c}</em>{rx} = \bar{c}_{rx} \left( L/B, \frac{a_0}{V_s} \right) \to \text{Graph c} )</td>
</tr>
<tr>
<td>Rocking ry</td>
<td>General ( K_{by} = \frac{G L}{V_s A_b} \left( \frac{L}{B} \right)^{0.5} \left( \frac{V_s}{a_0} \right)^{0.5} )</td>
<td>( k_y = 1 - 0.2 a_0 )</td>
</tr>
<tr>
<td>Torsion t</td>
<td>General ( K_t = G J L )</td>
<td>( C_t = (\rho V_s A) \bar{c}_t ) ( \bar{c}_t = \bar{c}_t \left( L/B, \frac{a_0}{V_s} \right) \to \text{Graph d} )</td>
</tr>
</tbody>
</table>

Table I

Homogeneous Halfspace

\( \rho, V, p \)

09/05/2012

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Effect of Bedrock at Shallow Depth

("Soil–Stratum Over Rock")
\[ K_v \approx \frac{4GR}{1-V^2} \left(1 + \frac{3R}{H} \right) \]
\[ K_h \approx \frac{8GR}{3-V} \left(1 + \frac{1}{2} \frac{R}{H} \right) \]
\[ K_r \approx \frac{8GR^3}{3(1-V)} \left(1 + \frac{1}{3} \frac{R}{H} \right) \]
\[ K_t \approx \frac{8GR^3}{3(1-V)} \left(1 + \frac{1}{12} \frac{R}{H} \right) \]
Surface Foundation
Homogeneous Stratum over Bedrock

Circular Foundation
Radius \(B = R\)

### Static vertical stiffness

\[
K_z = \frac{4GR}{1 - \nu^2} \left(1 + 1.3 \frac{R}{H}\right)
\]

### Dynamic stiffness coefficient

\[
k_z = k_z \left(\frac{H}{R}, a_0\right) \rightarrow \text{Graph III-1}
\]

### Radiation dashpot coefficient

\[
C_z \left(\frac{H}{B}\right) \approx 0, \quad f < f_c
\]

\[
C_z \left(\frac{H}{B}\right) \approx 0.8 C_z \left(\infty\right), \quad f \geq 1.5 f_c
\]

\[
f_c = \frac{V_{L_0}}{4H}, \quad V_{L_0} = \frac{3.4V_s}{\pi(1 - \nu)}
\]
\[ \mathcal{K}_Y = K_Y + i \omega C_Y \quad \bar{K}_Y = K_Y \bar{k}_Y(\omega) \]

Graphs Accompanying Table III

III-1 CIRCLE

III-2 RECTANGLE

III-3 STRIP

\[ k_y = k_z \]

\[ \frac{C_Y}{\rho V_S A} \]

\[ V_S / 4H \]

\[ V_S / 4H \]
CAISSON Foundations

PART C

Static and Dynamic Soil–CAISSON Interaction

(Linear and Non-Linear)
TYPES of FOUNDATION

PILES
L / d ≥ 7

CAISSONS
D / B ≈ 1 – 5

FOOTING
D / B ≤ 1

Examples of Caisson Foundations

Kobe

Port Island

TAGUS
Simple Methods Used in Practice

Specified Displacement Pattern

\[ \kappa = ? \]

Winkler

Elasticity

2 Ways of Viewing the CAISSON

1) as an Embedded Foundation

2) as a Large–Diameter Rigid Pile
the **CAISSON** as an

**Embedded Foundation**

*(Linear Response)*

---

### Foundation Embedded in Halfspace

<table>
<thead>
<tr>
<th></th>
<th>Static $K_{emb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VERTICAL Z</strong></td>
<td></td>
</tr>
<tr>
<td>$K_{z, emb}$</td>
<td>$K_{z, surf} \left[ \frac{1}{21} \frac{D}{B} (1 + 1.3 \chi) \right] \left[ 1 + 0.2 \left( \frac{A_x}{A_y} \right)^{2/3} \right]$</td>
</tr>
<tr>
<td>$K_{z, surf}$</td>
<td>$\frac{2GL}{1 - v} (0.73 + 1.54 \chi^{0.75})$</td>
</tr>
<tr>
<td>$\chi$</td>
<td>$A_y / 4 \ L^2$</td>
</tr>
</tbody>
</table>

| **HORIZONTAL Y** |                 |
| $K_{y, emb}$ | $K_{y, surf} \left[ 1 + 0.15 \frac{\sqrt{D} \ B}{B} \right] \left[ 1 + 0.52 \left( \frac{h}{B} \frac{A_y}{L^2} \right)^2 \right]$ |
| $K_{y, surf}$ | $\frac{2GL}{2 - v} (2 + 2.5 \chi^{0.85})$ |
| $\chi$ | $A_y / 4 \ L^2$ |

$A_W =$ area of surface in contact

Full Contact: $A_W = (4B + 4L)D$
# Partially/Fully Embedded Foundations

## Homogeneous Halfspace

<table>
<thead>
<tr>
<th>Vibration Mode</th>
<th>Dynamic Stiffness</th>
<th>Dynamic Stiffness</th>
<th>Radiation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( K_{emb}(\omega) = K_{emb} \times k_{emb}(\omega) )</td>
<td>( C_{emb}(\omega) = C_{emb} \times k_{emb}(\omega) )</td>
<td>( C_{emb}(\omega) = C_{surf} + p V_s A_{cw} )</td>
</tr>
<tr>
<td>Vertical z</td>
<td>( K_{z, emb} = K_{z, surf}[1 + \frac{1}{2}\left(\frac{D}{B}\right)[\frac{1 + 1.3 \times 0.2}{1 + 2}]} \times \left[1 + 0.2\left(\frac{D}{B}\right)^{2.5}\right] )</td>
<td>Fully embedded, ( a_0 \leq \frac{D}{B} \leq 2 )</td>
<td>General shape, ( C_{emb} = C_{surf} + p V_s A_{cw} )</td>
</tr>
<tr>
<td>Horizontal x, y</td>
<td>( K_{y, emb} = K_{y, surf}[1 + 0.15 \times \frac{L}{B}] \times \left[1 + 0.52\left(\frac{D}{B}\right)^{3.7}\right] )</td>
<td>Fully embedded, ( L/B = 6 )</td>
<td>Rectangular, ( C_{emb} = 4 p V_s B L C_{cw} + 4 p V_s (B + L) d )</td>
</tr>
</tbody>
</table>

\( a_0 = \frac{D}{B} \)

### Graphs Accompanying Table {Embedded}

![Graphs](image_url)
the CAISSON as a Large–Diameter Rigid Pile

(Non–Linear Response)
SOIL REACTIONS

PILE: only Lateral Stresses

CAISSON Lateral + Vertical Stresses

Soil – Caisson Interaction

A. Interface

B. Near Field

C. Far Field
EXAMPLES of STRUCTURES on CAISSON Foundations

Kobe Port Island

Soil Nonlinearity

Soil + Interface Nonlinearities

\[ \tau_{rz}, \sigma_r, \theta_c, u_x \]
SOIL REACTIONS

PILE:
only
Lateral
Stresses

CAISSON
Lateral
+ Vertical
Stresses

SOIL REACTION of CAISSON

Lateral Surface

\[ p_x(z) = \int_0^B \left[ \sigma_r \cos \psi + \tau_{rz} \sin \psi \right] r \, d\psi \]

\[ m_0(z) = \int_0^B \int_0^{2\pi} \left( \frac{B}{2} \right)^2 \sin \psi \, r \, d\psi \, dr \]

Section

\[ V_b = \int_0^B \int_0^{2\pi} \left[ -\tau_{rz} \cos \psi + \tau_{r\psi} \sin \psi \right] r \, d\psi \, dr \]

Base

\[ M_b = \int_0^B \int_0^{2\pi} \left( \sigma_z \cos \psi \right) r^2 \, d\psi \, dr \]
**LATERAL STATIC LOADING**

"Winkler" Model

---

**Horizontal Force Equilibrium:**

\[ V_0(t) - m \ddot{u}_c(t) - \int_0^D \tilde{k}_x(z) u(z,t) \, dz - \tilde{K}_H u_b(t) = 0 \]

**Moment Equilibrium Around Base** (\( z = 0 \))

\[ M_0(t) + V_0(t) D_{\alpha \dot{\alpha}} - J_c \ddot{\phi}_c(t) - m \dddot{u}_c(t) \frac{D_{\alpha \dot{\alpha}}}{2} - \int_0^D \tilde{k}_x(z) u(z,t) z \, dz + \int_0^D \tilde{k}_\theta(z) \phi(t) \, dz + \tilde{K}_M \phi(t) = 0 \]

**Force—Displacement Relation (Stiffness Matrix)**

\[
\begin{bmatrix}
V_{base} \\
M_{base}
\end{bmatrix} =
\begin{bmatrix}
\tilde{K}_{HH} - m \omega^2 & \tilde{K}_{HM} - m \omega^2 \frac{D_{\alpha \dot{\alpha}}}{2} \\
\tilde{K}_{HM} - m \omega^2 \frac{D_{\alpha \dot{\alpha}}}{2} & \tilde{K}_{MM} - J_c \omega^2 - m \omega^2 \frac{D_{\alpha \dot{\alpha}}^2}{4}
\end{bmatrix}
\begin{bmatrix}
u_b \\
\phi_b
\end{bmatrix}
\]
**STATIC SPRING CALIBRATION**

**BASE SPRINGS**

\[
K_N = \frac{4Gb}{2-v} \quad K_M = \frac{8Gb^3}{3(1-v)}
\]

**DISTRIBUTED SPRINGS**

**EMBEDDED FOUNDATION THEORY**  
(Gazetas et al, 1987, 1989)

\[
\begin{align*}
\frac{k_x}{E_s} &= f(D,B) \\
\frac{k_\theta}{E_s B^2} &= f(D,B)
\end{align*}
\]

---

**Analytical Relations for Elastic Springs**  
(Gerolymos & Gazetas, 2000)

**Square Caisson**

\[
k_x = 1.68 \left(\frac{D}{B}\right)^{-0.13} E_s \quad k_\theta = 1.23 \left(\frac{D}{B}\right)^{-1.313} E_s D^2
\]

**Circular Caisson**

\[
k_x = 1.37 \left(\frac{D}{B}\right)^{-0.13} E_s \quad k_\theta = 0.9 \left(\frac{D}{B}\right)^{-1.477} E_s D^2
\]
Soil – Caisson Interaction

A. Interface

B. Near Field

C. Far Field

“Winkler” Model

BWGG for Caissons
Medium—Scale Field Load Test of Caisson
(EPRI, 1981)

<table>
<thead>
<tr>
<th>Thick.</th>
<th>γ</th>
<th>$S_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.68</td>
<td>20.6</td>
<td>75</td>
</tr>
<tr>
<td>1.38</td>
<td>22</td>
<td>100</td>
</tr>
<tr>
<td>0.76</td>
<td>22</td>
<td>170</td>
</tr>
</tbody>
</table>

Static Loading of Caisson: $B = 1.5\,\text{m}$, $D = 4\,\text{m}$, in Clay

$M : \text{MNm}$

BWGG

Recorded
(EPRI, 1981)
Soil Reaction $p: \text{kN/m}$

**Diagram:**
- Soil reaction $p$ vs. depth $z$ in meters.
- Recorded for $0.6M_u$ (EPRI, 1981).
- BWGG

**Dynamic Field Load Test of Caisson**

**Equation:**
$$M = 2000 \sin (12t)$$

**Layer Details:**
- **Clay**:
  - Depth: 4.1 m
  - Thickness: 0.3 m
  - $\gamma$: 1.68
  - $S_u$: 75 kPa

- **Clay**:
  - Depth: 5.62 m
  - Thickness: 1.38 m
  - $\gamma$: 22
  - $S_u$: 100 kPa

- **Soft Schist**:
  - Depth: 7.14 m
  - Thickness: 0.76 m
  - $\gamma$: 22
  - $S_u$: 170 kPa
Kinematic Response
excitation: Sepolia (Athens 1999)
Batter Piles under Seismic Loading

3D-FE Modelling

- Group with vertical piles
- Asymmetric group with inclined pile
- Symmetric group with inclined piles
3. Loading

Kinematic Loading + Inertial Loading

Normalized Maximum Bending Moment with respect to the group of fixed-head vertical piles – Lefkada

Fixed

Hinged

Vertical Pile
Inclined Pile
Inclined Pile
Normalized Maximum Bending Moment of the Pile with respect to the Fixed-Head Vertical Pile - Lefkada

Normalized Maximum Bending Moment of the Pile with respect to the Fixed-Head Vertical Pile - Lefkada

Spectral Accelerations at the Deck

Spectral Accelerations at the Deck

09/05/2012